OBJECTIVES

1. **Given a mathematical expression, identify the variables.**
2. **Translate simple word problems into mathematical symbols and solve.**

SECTION 1.1

Numbers and Algebra

Identifying Variables

One of the first mathematical concepts we learned as children was the association between objects and numbers. The numbers used in this association were 1, 2, 3, 4, and so forth, which are called **natural** or **counting numbers**. When the number 0 is included, we call them **whole numbers**.

Later we learned another association with numbers: The word *one* meant the symbol 1 in mathematics, *two* meant 2, and so on. Symbols that represent numbers are called **constants** because they have a fixed, or constant, value. For instance, 1, 2, $\frac{1}{2}$, $\frac{5}{8}$, and so on are constants.

We learned that words like *add*, *subtract*, *multiply*, and *divide* describe some **operation** between numbers or objects. With these operations, we learned what $2 + 9$, $7 - 4$, 3×6, and $12 \div 6$ meant. We learned, for example, that $2 + 9$ was another way of writing 11. And we learned to use the symbol $=$ to mean equality $(2 + 9 = 11)$. We thus formed an **equation**.

An equation is a mathematical statement that two quantities are equal.

We use these four operations in algebra, along with a mathematical alphabet that consists of the letters $a, b, c, \ldots, x, y, z$. When a letter can be replaced by different numbers, we call that letter a **variable**. For instance, if the letter p represents any number, then the sum of p and 5 is written $p + 5$. We can replace p by 1 to get $1 + 5$, or by 2 to get $2 + 5$, and so forth. The letter p is a variable; the numbers 1, 2, and 5 are constants. Variables are not restricted to the English alphabet. If this book were written in Spanish, Arabic, Chinese, Russian, or any other language, then alphabetic symbols from those languages would be used as variables instead.

In English sentences, variables are either pronouns or impersonal nouns that do not have an exact meaning. However, they can be given an exact meaning by replacing the nonspecific word with a specific one, say a person's name. For instance, in the sentence, "She likes studying math," the word *she* acts as a variable. *She* can refer to any female person. If we replace *she* by *Maria*, then we have the statement, "Maria likes studying math." *Maria* acts as a constant (in the mathematical sense).

Just as the alphabet is used to build words and words are used to make sentences, variables—when combined with operation symbols and other variables or constants—make mathematical sentences, or **expressions**. Consider the following examples, in which we identify the variables in a mathematical expression.

EXAMPLE 1

$$x + y + z$$

Solution

The variables are x, y, and z.

EXAMPLE 2

$$p - 3$$

Solution

The variable is p.

EXAMPLE 3

$$\frac{m}{n}$$

Solution

The variables are m and n.

EXAMPLE 4

$$r^2 - 2rs + s^2$$

Solution

The variables are r and s.

EXAMPLE 5

$$(2 - 1)(5 + 2)$$

Solution

There are no variables.

SELF-CHECK

Identify the variables in the following mathematical expressions:

1. $t - u$ ________ and ________ **2.** $m^3 - n^3$ ________ and ________

3. $\frac{3}{z}$ ________ **4.** $x^2 - 2x + 1$ ________

Please check your answers against those given at the bottom of this page.

Identifying a variable in a mathematical expression is easy. You simply read from left to right and look for all the different letters that appear.

Translating Word Problems

One part of the process of translating word problems into mathematical expressions or equations is searching for words that relate to mathematical operations. In Table 1.1 we have listed the operations, their associated mathematical symbol, and the related words most often used in word problems. Use the table as a guide whenever necessary.

OPERATION	OPERATION SYMBOL	RELATED WORDS
Addition	$+$	add, added to, and, increased by, increment, plus, sum, the sum of, total
Subtraction	$-$	decreased by, difference, difference of, difference between, subtract, subtract from, take away
Multiplication	$\cdot$ $\times$ ( )( ) [ ][ ] { }{ }	multiply, multiplied by, product, the product of, times, of, fraction to percent
Division	/ — $\div$	divide, divided by, quotient, remainder

TABLE 1.1

ANSWERS TO SELF-CHECK

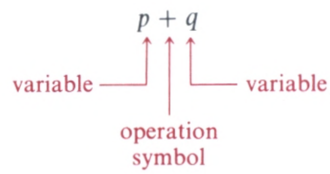
1. t, u **2.** m, n **3.** z **4.** x

EXAMPLE 6

Write an expression that illustrates the sum of two numbers using variables.

Solution

Looking at Table 1.1, we find the word *sum* and the corresponding operation symbol. The symbol we use is $+$. Since we want to write an expression that illustrates the sum of two numbers, we arbitrarily choose two variables, say p and q. We then write



So, one possible expression that illustrates the sum of two numbers is: $p + q$.

REMARK

In Example 6 $x + y$ would be just as correct as $p + q$ because we can arbitrarily choose the variables we want to use. Notice that operation symbols are placed between expressions or variables. Thus, we cannot write $pq+$ or $+pq$ to mean the sum of two numbers.

EXAMPLE 7

Write an expression that illustrates:

a. the difference of two numbers

b. the product of two numbers

c. the quotient of two numbers

Use the variables m and n.

Solution

a. The operation symbol for difference is $-$, so we write $m - n$.

b. There are many ways to write the product of two numbers. Referring to Table 1.1, we see that $m \cdot n$, $(m)(n)$, $[m][n]$, and $\{m\}\{n\}$ are four different ways to express the product, but mn is the most common.

c. Two ways to express the quotient are: $\frac{m}{n}$ and $m \div n$.

Grouping Symbols

The symbols ( ), [ ], and { } are called **grouping symbols**. They are used to group expressions together so we can think of them as one quantity. In Example 7b, these grouping symbols surrounded quantities that happened to be variables. Any variable or number can be enclosed with grouping symbols. Thus, we can write $x + y$ as $(x) + (y)$

$$\text{or} \quad \frac{x}{y} \quad \text{as} \quad \left[\frac{x}{y}\right]$$

$$\text{or} \quad 2 + 3 \quad \text{as} \quad [2] + (3)$$

These are all acceptable expressions in algebra. Also, notice that these grouping symbols *always* come in pairs. For every left parenthesis, for example, there is a right parenthesis.

When a grouping symbol surrounds more than one variable or number, we begin to read the expression by using the phrase "the quantity . . . ," as illustrated in the next example.

EXAMPLE 8

Write the following mathematical expressions in English:

a. $(2 + 3)/5$ **b.** $(x + y)(x - y)$ **c.** $a \cdot (b + c)$

Solution

a. In the expression $(2 + 3)/5$ we first notice that parentheses are used to enclose $2 + 3$. Next we notice the symbol /, which is used to denote division. Finally, we see a 5. We read $(2 + 3)/5$ as "the quantity two plus three divided by five."

b. $(x + y)(x - y)$ is an expression consisting of two quantities. The first quantity is $x + y$ and the second is $x - y$. We are multiplying these two quantities. We read the expression as "the quantity x plus y times the quantity x minus y."

c. We read $a \cdot (b + c)$ as "a times the quantity b plus c."

REMARK

When parentheses ( ), brackets [ ], or braces { } are written "back to back" as in any of the following:

$$)(\quad][\quad \}\{ \quad](\quad \}(\quad)[\quad)\{ \quad \}[\quad]\{$$

the operation is always multiplication.

For instance, 2 times 3 can be written in any of the following ways:

$$(2)(3) \quad [2][3] \quad \{2\}\{3\} \quad [2](3) \quad \{2\}(3)$$
$$(2)[3] \quad (2)\{3\} \quad \{2\}[3] \quad [2]\{3\}$$

SELF-CHECK

Translate the given mathematical expressions into English.

5. $2/(3 + 5)$ ____________________

6. $\dfrac{(x - 1)}{(x + 1)}$ ____________________

In mathematics the positioning of symbols is very important. If we compare Example 8a and Self-Check problem 5, we see that the numbers 2, 3, and 5 are identical but the remaining symbols / and ( ) are used differently. These two problems have different meanings and different English translations. In the example that follows we translate an English phrase into mathematical symbols.

EXAMPLE 9

Write an expression that illustrates the following:

The sum of p and q is added to the product of r and s.

ANSWERS TO SELF-CHECK

5. two divided by the quantity three plus five

6. the quantity x minus one divided by the quantity x plus one

Solution

In Table 1.1, we find several key words that appear in this phrase: *sum*, *added to*, and *product*.

"The sum of p and q" is written	$p + q$
"The product of r and s" is written	rs

The final answer is: $(rs) + (p + q)$, or, in shorter form, $rs + p + q$.

SELF-CHECK

Write a mathematical expression to illustrate the following:

7. Subtract the product of s and t from the sum of p and q.

8. From the difference of r and s add the sum of p and 3.

When we translate an English sentence into a mathematical sentence, we should watch for words or phrases like *equal*, *is equal to*, *gives*, *results*, and so forth. These usually denote equality and are represented by the symbol = in mathematics.

EXAMPLE 10

Write a mathematical sentence that illustrates the following:

When three is added to some number, the result is nine.

Solution

Reading from left to right we look for words that can be translated into mathematical symbols.

Three	3	
Is added to	+	
Some number	m	an arbitrary variable we can choose
The result is	=	
Nine	9	

Since 3 is added to m, we write $m + 3$. The final sentence is $m + 3 = 9$.

SELF-CHECK

Write a mathematical sentence to illustrate the following:

9. If six is subtracted from some number, the result is ten.

10. Seven times some number gives fifty-six.

Finding an answer to a word problem is sometimes tricky because we must determine what important information is contained in the problem, how the information is related to the final answer, what the relationships are between the pieces of information, and so forth. Always read through the entire problem first, then reread looking for key words. Make sure you understand what the given information is and what you are asked to find.

ANSWERS TO SELF-CHECK

7. $(p + q) - st$ **8.** $(r - s) + (p + 3)$
9. $x - 6 = 10$ **10.** $7p = 56$

EXAMPLE 11

According to the Office of Management and Budget (OMB), a family of two that makes less than \$4,500 is considered poor. For each additional family member OMB adds \$1,100. What would OMB consider poor for a family of three?

Solution

At this point, we have already read the problem once. Now we reread to search for key words. The only key word is *adds*. Reading a third time, we search for the given information and what we must find. We could say the first two sentences are the given and the final sentence is what we must find. However, the first two sentences contain information that is unnecessary to solving the problem, for instance what the OMB is. The only important pieces of information we need to know to solve the problem are:

Known: Family of two making less than \$4,500 is poor. Add \$1,100 for each additional family member.

Unknown: What OMB considers poor for a family of three.

We next form an expression relating the known information:

$$\underbrace{\$4{,}500}_{\text{two people}} + \underbrace{\$1{,}100}_{\text{additional family member}}$$

Adding these two amounts gives the answer: \$5,600. So, if a family of three has an income of \$5,600 or less, OMB considers that family poor.